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Time-domain analysis of one-dimensional electromagnetic scattering by lossy media

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Institute : TNO Physics and Electronics Laboratory

Date : October 1990

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Research supervised by : Ir C.D. de Haan

Research carried out by : Ir J.J.A. Klaasen

ABSTRACT (UNCLASSIFIED)

A study was performed to investigate the electromagnetic scattering in the time domain by a plane interface between two half-spaces. One half-space is assumed to be vacuum, while the other half-space is homogeneous and consists of lossy material. The incident field is assumed to be a uniform plane wave. Hence, this study addresses the one-dimensional scattering problem.
Both horizontal and vertical polarization of the incident field are addressed.
Starting with the equations for the reflected and transmitted waves in the s- or Laplace-domain, corresponding time-domain expressions are obtained by applying the inverse Laplace transform analytically. These time-domain results are in closed form, i.e. are given in terms of elementary functions or integrals of elementary functions.
Numerical results are presented for the scattering of a unit-step and a Nuclear ElectroMagnetic Pulse (NEMP), using the derived time-domain expressions. The numerical implementation uses a time-marching procedure.

Rapport no. : FEL-90-A211
Titel : Tijddomein analyse van één-dimensionale verstrooiing door een medium met verliezen

Auteur(s) : Ir J.J.A. Klaasen
Instituut : Fysisch en Elektronisch Laboratorium TNO
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Onderzoek uitgevoerd o.l.v. : Ir C.D. de Haan
Onderzoek uitgevoerd door : Ir J.J.A. Klaasen

SAMENVATTING (ONGERUBRICEERD)

Een studie werd verricht naar de elektromagnetische verstrooiing in het tijddomein aan een grensvlak tussen twee half-ruimten. Eén van de half-ruimten bestaat uit vacuum, terwijl de andere half-ruimte uit homogeen materiaal bestaat, welke verliezen vertoont. Voor deze studie wordt er vanuit gegaan dat het invallende veld een uniforme vlakke golf is. Dus, deze studie behandelt het een-dimensionale verstrooiingsprobleem.
Zowel horizontaal als verticaal gepolariseerde invallende velden worden beschouwd.
Uitgaande van de uitdrukkingen voor de gereflecteerde en doorgelaten golven in het s- of Laplace-domein, worden de overeenkomstige tijddomein uitdrukkingen verkregen na het analytisch toepassen van de inverse Laplace transformatie. Deze tijddomein uitdrukkingen kunnen worden geschreven als elementaire functies of integralen hiervan.
Numerieke resultaten worden gepresenteerd van de verstrooiing van een eenheidsstap en een Nucleaire ElektroMagnetische Puls (NEMP) met behulp van de verkregen tijddomein uitdrukkingen. De numerieke implementatie maakt gebruik van een time-marching procedure.

1 INTRODUCTION

In most electromagnetic scattering configurations the influence of the earth has to be taken into account, because an object in the vicinity of the earth's surface is not only irradiated by the incident field, but also by the field reflected by the earth. Such scattering configurations occur in Nuclear ElectroMagnetic Pulse (NEMP) interaction studies, which are conducted by the EMP-group of the Physics and Electronics Laboratory of TNO (FEL-TNO). Therefore, a study was performed to investigate the electromagnetic scattering by a plane interface between two half-spaces. One of the half-spaces consists of vacuum, and the other half-space consists of homogeneous and lossy material. In a previous study this scattering problem has been solved in the Laplace- or s-domain, Klaasen [1], but in the present study, of which this report presents the results, it is solved in the time domain.
Since the incident field is assumed to be a uniform plane wave, this study addresses the one-dimensional scattering problem.

Throughout this report it is presumed that the reader has some basic knowledge of system theory, since this is not exemplified in the text. For a comprehensive treatise of system theory the reader is referred to Papoulis [2].

One of the most well-known methods to obtain time-domain results, is to compute the response of a configuration for a large number of frequencies, after which a numerical inverse Fast Fourier Transform (FFT) is applied. In this report *exact* analytical expressions in the time domain are obtained by applying an *analytical* inverse Laplace-transform to the Laplace-domain expressions for the reflected and transmitted electromagnetic fields. The waveform of the incident field is taken as a unit-step function. The then obtained unit-step response is in closed form, i.e. is given in terms of elementary functions or

integrals of elementary functions.
In Appendix A a method is presented to obtain the response to any other waveform from the unit-step response. This method typically involves a kind of convolution integral.

In Chapter 2 the scattering configuration is presented and the solution of the scattering problem is presented in the Laplace domain, for both vertically and horizontally polarized incident fields. In Chapter 3 and Chapter 4 the Laplace-domain expressions of Chapter 2 are transformed back to the time domain for the vertically and horizontally polarized incident fields, respectively. To illustrate the outlined procedure, the reflected and transmitted electromagnetic fields of a unit-step function and a NEMP are determined in Chapter 5. Finally, conclusions are drawn in Chapter 6.

2 DESCRIPTION AND SOLUTION OF THE ELECTROMAGNETIC SCATTERING PROBLEM IN THE LAPLACE DOMAIN

In Section 2.1 the scattering configuration is described, and definitions and conventions are defined. In Sections 2.2 and 2.3 the solution of the scattering problem is given in the Laplace domain. In later chapters the expressions given in these two sections are used to obtain the time-domain expressions.
For a more detailed treatment of the electromagnetic scattering in the Laplace domain the reader is referred to Klaasen [1].

2.1 Description of the scattering configuration

The onedimensional scattering configuration depicted in Fig. 2.1 is considered. To specify the position in this unbounded space, we employ the coordinates {x, y, z} with respect to a given orthogonal right-handed Cartesian reference frame with origin O. The unit vectors along the axes are denoted by $\underline{i}_x$, $\underline{i}_y$ and $\underline{i}_z$. Then the position vector in this coordinate system is given by

$$\underline{r} = x\underline{i}_x + y\underline{i}_y + z\underline{i}_z. \tag{2.1}$$

The time coordinate is denoted by t. Let the Laplace transform of $f(t)$ be denoted by $\hat{f}(s)$. Then $f(t)$ and $\hat{f}(s)$ are related by the Laplace transform with respect to time, defined by

$$\mathcal{L}\{f(t)\} = \hat{f}(s) = \int_0^\infty e^{-st} f(t)\, dt, \tag{2.2}$$

while the inverse Laplace transform is given by

$$f(t) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} e^{st}\, \hat{f}(s)\, ds, \tag{2.3}$$

where i is the imaginary unit, and s denotes the complex frequency. The integration for the inverse Laplace transform is performed along the Bromwitch contour, LePage [4], with α a positive real number.

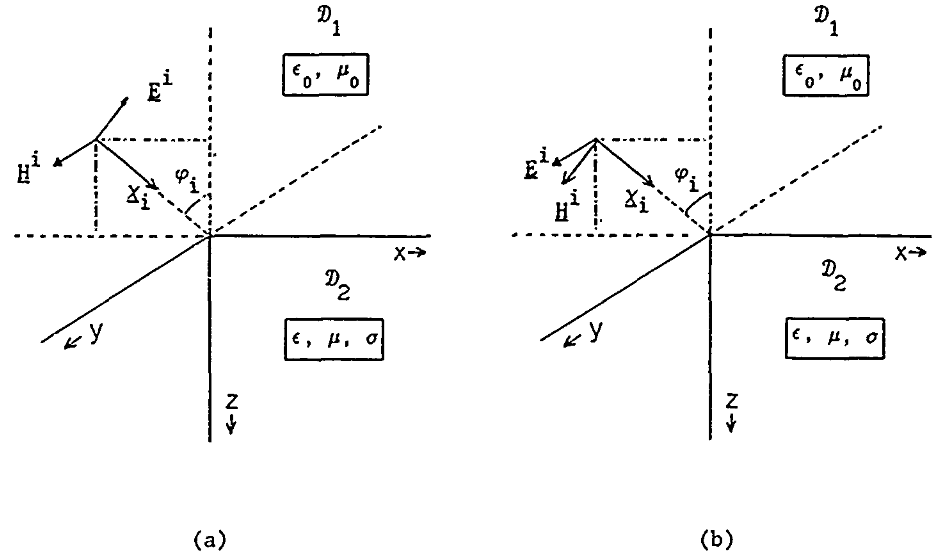


Fig. 2.1. Electromagnetic scattering configuration.
a) vertically polarized field,
b) horizontally polarized field.

Let the half-space $\mathcal{D}_1$ be given by

$$(2.4) \qquad \mathcal{D}_1 = \{ x, y, z \in \mathbb{R} \mid z < 0 \},$$

and let the half-space $\mathcal{D}_2$ be given by

$$(2.5) \qquad \mathcal{D}_2 = \{ x, y, z \in \mathbb{R} \mid z > 0 \}.$$

This configuration is irradiated by an incident uniform electromagnetic plane wave. The source, which generates the incident field, is located in $\mathcal{D}_1$. The constitutive constants of the half-space $\mathcal{D}_1$ are assumed to be those of vacuum, i.e. ϵ_0 and μ_0. The half-space $\mathcal{D}_2$ is assumed to consist of homogeneous, isotropic, time-invariant, linear and instantaneous and locally reacting material. $\mathcal{D}_2$ is characterized in its electromagnetic behavior by the permittivity ϵ, the permeability μ, and the conductivity σ. Furthermore, there are no sources present in $\mathcal{D}_2$.

The incident wave propagates in the direction of the propagation vector $\underline{\chi}_i$. The propagation vector $\underline{\chi}_i$ is of unit length, i.e. $\|\underline{\chi}_i\| = 1$, and makes an angle φ_i with the normal $\underline{n}$ on the plane interface between the two media, with $\varphi_i < 90^\circ$. The vector $\underline{n}$ is also of unit length and is directed inwards $\mathcal{D}_2$.

This configuration allows wave propagation in two independent ways : vertically and horizontally polarized incident fields. These two ways of wave propagation are addressed in the next two successive sections.

2.2 Vertically polarized incident field

Let the incident magnetic field be a uniform plane wave, and let it have an y-component only. Furthermore, the incident field hits the interface between $\mathcal{D}_1$ and $\mathcal{D}_2$ at t = 0. Then the incident magnetic field is given by

$$(2.6) \qquad \hat{H}^i_y(\underline{r},s) = \hat{H}_0(s)\, e^{-\gamma_0(\underline{\chi}_i \cdot \underline{r})}, \qquad \underline{r} \in \mathcal{D}_1$$

with the propagation vector $\underline{\chi}_i$ given by

$$(2.7) \qquad \underline{\chi}_i = \sin\varphi_i \underline{i}_x + \cos\varphi_i \underline{i}_z .$$

In eq.(2.6), the propagation constant γ_0 is given by

$$(2.8) \qquad \gamma_0 = s\,(\epsilon_0\mu_0)^{1/2} = \frac{s}{c_0},$$

where c_0 denotes the speed of light in vacuum. $\hat{H}_0(s)$ denotes the Laplace transform of the waveform of the incident magnetic field.
The incident electric field follows from Maxwell's equations

$$(2.9) \qquad \begin{aligned} \hat{E}^i_x(\underline{r},s) &= \cos\varphi_i\, Z_0\hat{H}^i_y, \\ \hat{E}^i_z(\underline{r},s) &= -\sin\varphi_i\, Z_0\hat{H}^i_y, \end{aligned} \qquad \underline{r} \in \mathcal{D}_1$$

where Z_0 denotes the wave impedance of vacuum given by

$$(2.10) \qquad Z_0 = \hat{E}_0 / \hat{H}_0 = \left[\frac{\mu_0}{\epsilon_0}\right]^{1/2}.$$

The propagation constant of the transmitted fields in $\mathcal{D}_2$ is

$$(2.11) \qquad \gamma = \|\underline{\gamma}\| = (s\mu(s\epsilon + \sigma))^{1/2},$$

where $\underline{\gamma}$ is given by

$$(2.12) \qquad \underline{\gamma} = \gamma_x\underline{i}_x + \gamma_z\underline{i}_z,$$

with

$$(2.13) \qquad \begin{aligned} \gamma_x &= \gamma_0\sin\varphi_i, \\ \gamma_z &= \sqrt{\gamma^2-\gamma_x^2}, \qquad \mathrm{Re}(\gamma_z) \geq 0. \end{aligned}$$

The transmitted wave in $\mathcal{D}_2$ must obey the radiation condition at $z \to \infty$.

Therefore, we must select that branch of γ_z where $\mathrm{Re}(\gamma_z)$ is positive. Now let v denote the propagation speed of the waves in $\mathcal{D}_2$ in the lossless case, then

$$(2.14) \qquad v = \frac{1}{\sqrt{\epsilon\mu}}.$$

Subsequently, we define φ_t as the angle of refraction, also in the lossless case. Hence,

$$(2.15) \qquad \cos\varphi_t = \sqrt{1 - \frac{v^2}{c_0^2}\sin^2\varphi_i},$$

which follows directly from Snell's law.
Using eqs.(2.14)-(2.15), the propagation constant γ_z is represented as

$$(2.16) \qquad \gamma_z = \frac{\cos\varphi_t}{v}\sqrt{s^2 + 2as},$$

where

$$(2.17) \qquad a = \frac{\sigma}{2\epsilon}\cos^{-2}\varphi_t.$$

Using eq.(2.14)-(2.17), the transmitted fields in $\mathcal{D}_2$ can be expressed as

$$(2.18) \qquad \begin{aligned} \hat{H}_y^t(\underline{r},s) &= \hat{T}^H\hat{H}_0 e^{-(\underline{\gamma}\cdot\underline{r})} = \hat{T}^H\hat{H}_0 e^{-s\frac{x}{v}\sin\varphi_t - \frac{z}{v}\cos\varphi_t\sqrt{s^2+2as}}, \\ \hat{E}_x^t(\underline{r},s) &= Z_t\hat{H}_y^t, \qquad\qquad \underline{r}\in\mathcal{D}_2 \\ \hat{E}_z^t(\underline{r},s) &= -Z_n\hat{H}_y^t, \end{aligned}$$

where Z_n and Z_t denote the normal and transverse (or surface) wave impedances, respectively, given by

$$Z_t = \frac{\gamma_z}{s\epsilon + \sigma},$$

(2.19)

$$Z_n = \frac{\gamma_x}{s\epsilon + \sigma}.$$

$\hat{T}^H$ in eq.(2.18) denotes the transmission coefficients for vertical polarization and is represented by

$$\hat{T}^H = \frac{2(s + \frac{\sigma}{\epsilon})}{s + \frac{\sigma}{\epsilon} + \nu\sqrt{s^2 + 2as}}, \tag{2.20}$$

where

$$\nu = \sqrt{\mu_r/\epsilon_r}\,\frac{\cos\varphi_t}{\cos\varphi_i}. \tag{2.21}$$

The reflected fields in $\mathcal{D}_1$ propagate in the direction of the propagation vector $\underline{\chi}_r = \sin\varphi_i \underline{i}_x - \cos\varphi_i \underline{i}_z$, and are represented by

$$\hat{H}^r_y(\underline{r},s) = \hat{R}^H \hat{H}_0 e^{-\gamma_0(\underline{\chi}_r\cdot\underline{r})},$$

(2.22)

$$\hat{E}^r_x(\underline{r},s) = -\cos\varphi_i\, Z_0\hat{H}^r_y, \qquad \underline{r} \in \mathcal{D}_1$$

$$\hat{E}^r_z(\underline{r},s) = -\sin\varphi_i\, Z_0\hat{H}^r_y,$$

where $\hat{R}^H$ denotes the reflection coefficient for vertical polarization.

$\hat{R}^H$ is given by

$$\hat{R}^H = \frac{s + \frac{\sigma}{\epsilon} - \nu\sqrt{s^2 + 2as}}{s + \frac{\sigma}{\epsilon} + \nu\sqrt{s^2 + 2as}}. \tag{2.23}$$

2.3 Horizontally polarized incident field

Let the incident electric field be a uniform plane wave. Subsequently, let it have an y-component only. Again, the incident field hits the interface between $\mathcal{D}_1$ and $\mathcal{D}_2$ at $t = 0$. Then the incident electric field is represented by

$$\hat{E}^i_y(\underline{r},s) = \hat{E}_0(s)\, e^{-\gamma_0(\underline{\chi}_i \cdot \underline{r})}, \qquad \underline{r} \in \mathcal{D}_1 \tag{2.24}$$

with $\underline{\chi}_i$ given by eq.(2.7), and γ_0 given by eq.(2.8). $\hat{E}_0(s)$ denotes the Laplace transform of the waveform of the incident electric field. From Maxwell's equations, we obtain for the incident magnetic field

$$\begin{aligned} \hat{H}^i_x(\underline{r},s) &= -\cos\varphi_i\, Y_0 \hat{E}^i_y, \\ \hat{H}^i_z(\underline{r},s) &= \sin\varphi_i\, Y_0 \hat{E}^i_y, \end{aligned} \qquad \underline{r} \in \mathcal{D}_1 \tag{2.25}$$

where Y_0 denotes the wave admittance of vacuum given by

$$Y_0 = \hat{H}_0 / \hat{E}_0 = \left[\frac{\epsilon_0}{\mu_0}\right]^{1/2}. \tag{2.26}$$

Then, the transmitted waves can be expressed as

$$
\begin{aligned}
&\hat{E}^t_y(\underline{r},s) = \hat{T}^E \hat{E}_0 e^{-(\underline{\gamma}\cdot\underline{r})}, \\
(2.27)\qquad &\hat{H}^t_x(\underline{r},s) = -Y_t \hat{E}^t_y, \qquad\qquad \underline{r} \in \mathcal{D}_2 \\
&\hat{H}^t_z(\underline{r},s) = Y_n \hat{E}^t_y,
\end{aligned}
$$

and the reflected waves in $\mathcal{D}_1$ are given by

$$
\begin{aligned}
&\hat{E}^r_y(\underline{r},s) = \hat{R}^E \hat{E}_0 e^{-\gamma_0(\underline{\chi}_r\cdot\underline{r})}, \\
(2.28)\qquad &\hat{H}^r_x(\underline{r},s) = \cos\varphi_i \, Y_0 \hat{E}^r_y, \qquad\qquad \underline{r} \in \mathcal{D}_1 \\
&\hat{H}^r_z(\underline{r},s) = \sin\varphi_i \, Y_0 \hat{E}^r_y,
\end{aligned}
$$

where Y_n and Y_t denote the normal and transverse (or surface) wave admittances, respectively, given by

$$
\begin{aligned}
&Y_t = \frac{\gamma_z}{s\mu}, \\
(2.29)\qquad & \\
&Y_n = \frac{\gamma_x}{s\mu}.
\end{aligned}
$$

In eq.(2.27) and (2.28) $\hat{R}^E$ and $\hat{T}^E$ denote the reflection and transmission

coefficient for horizontally polarization, respectively, given by

$$\hat{R}^E = \frac{s - \rho\sqrt{s^2 + 2as}}{s + \rho\sqrt{s^2 + 2as}},$$

(2.30)

$$\hat{T}^E = \frac{2s}{s + \rho\sqrt{s^2 + 2as}},$$

where

$$\rho = \sqrt{\epsilon_r/\mu_r}\,\frac{\cos\varphi_t}{\cos\varphi_i}. \tag{2.31}$$

3 TIME-DOMAIN SOLUTION FOR A VERTICALLY POLARIZED INCIDENT FIELD WITH A UNIT-STEP WAVEFORM

The Laplace-domain expressions for the transmitted and reflected fields derived in Section 2.2, are used to obtain the unit-step response in the time domain. This is done by applying the inverse Laplace transform to these expressions analytically. How to obtain the response to other waveforms than the unit-step, which is denoted by U(t), is described in Appendix A.

3.1 Transmitted magnetic field

The transmitted fields for the vertical polarization are given by eq.(2.18) together with (2.20). To derive the unit-step response $\hat{H}_0$ is taken as s^{-1}, since the Laplace transform of the unit-step function $\mathcal{L}\{U(t)\} = s^{-1}$. The transmitted H-field is then rewritten as

$$\hat{H}_y^t = \frac{2(s + \frac{\sigma}{\epsilon})}{s + \frac{\sigma}{\epsilon} + \nu\sqrt{s^2 + 2as}}\, s^{-1}\, e^{-s \frac{x}{v} \sin \varphi_t - \frac{z}{v} \cos \varphi_t \sqrt{s^2 + 2as}}, \tag{3.1}$$

with ν given by eq.(2.21).
It can be shown that if the angle of incidence φ_i equals the Brewster angle φ_b^H (see Stratton [3]) $\nu = 1$. The Brewster angle for vertical polarization can be found from

$$\tan \varphi_b^H = \left[\frac{\epsilon_r(\epsilon_r - \mu_r)}{\epsilon_r\mu_r - 1}\right]^{1/2}. \qquad \epsilon_r \geq \mu_r \tag{3.2}$$

Furthermore, $0 < \nu < 1$ if $\varphi_i < \varphi_b^H$, and $\nu > 1$ if $\varphi_i > \varphi_b^H$.
After rearranging terms and multiplying the denominator as well as the

numerator of eq.(3.1) by $(s + \frac{\sigma}{\epsilon}) - \nu\sqrt{s^2 + 2as}$ (notice that this term equals the numerator of the reflection coefficient $\hat{R}^H$), we obtain

$$\text{(3.3)} \qquad \hat{H}_y^t = 2\, e^{-s \frac{x}{v} \sin \varphi_t} \left(1 + \frac{\sigma}{s\epsilon}\right) \frac{(s + \frac{\sigma}{\epsilon}) - \nu\sqrt{s^2 + 2as}}{(s + \frac{\sigma}{\epsilon})^2 - \nu^2(s^2 + 2as)} \times$$

$$e^{-\frac{z}{v} \cos \varphi_t \sqrt{s^2 + 2as}} .$$

The poles and zero of the transmitted magnetic field are the roots of the equation

$$\text{(3.4)} \qquad (1-\nu^2)s^2 + 2a(2\cos^2\varphi_t - \nu^2)s + (\frac{\sigma}{\epsilon})^2 = 0,$$

and are found to be (for $\nu \neq 1$)

$$\text{(3.5)} \qquad \begin{aligned} s_1 &= -\frac{a}{1-\nu^2}\left(2\cos^2\varphi_t - \nu^2 + \nu\sqrt{\nu^2 - \sin^2 2\varphi_t}\right), \\ s_2 &= -\frac{a}{1-\nu^2}\left(2\cos^2\varphi_t - \nu^2 - \nu\sqrt{\nu^2 - \sin^2 2\varphi_t}\right). \end{aligned}$$

Observe that the term under the root is always larger than or equal to zero, so that s_1 and s_2 are real valued.
Inspection of eqs.(3.3) and (3.5) shows that the denominator of the transmission coefficient has zeros given by s_1 and s_2. However, if $\nu \geq 1$ the numerator has a zero given by s_1. Therefore, if $\nu < 1$ the transmission coefficient has two poles given by s_1 and s_2, and if $\nu \geq 1$ it has only one pole given by s_2. This is because the zeros of the numerator and the denominator, both given by s_1, cancel each other.
If there are two poles (i.e. $\nu < 1$), they are in general different. But

if $\varphi_i = \pm 45^0$ and $\mu_r = 1$ they are the same, because then the term under the root vanishes.
From all the expressions derived in Chapter 2, we observe that the reflected and transmitted waves can be expressed in terms of the functions $\hat{g}_0$ and $\hat{g}_1$ defined as

$$\hat{g}_0(s,\xi_1,\xi_2,z) = \frac{s(s+2)}{(s+\xi_1)(s+\xi_2)} \frac{e^{-z\left[\sqrt{s^2+2s}-(s+1)\right]}}{\sqrt{s^2+2s}},$$

(3.6)

$$\hat{g}_1(s,\xi_1,\xi_2,\lambda,z) = \frac{(s+\lambda)}{(s+\xi_1)(s+\xi_2)} e^{-z\left[\sqrt{s^2+2s}-(s+1)\right]},$$

where ξ_1, ξ_2 and λ are parameters later to be determined.
Using the just defined functions $\hat{g}_0$ and $\hat{g}_1$ the expression for the transmitted magnetic field can be written as

$$\hat{H}^t_y = \frac{2}{1-\nu^2} e^{-z/\delta} e^{-\frac{s}{v}(\underline{\chi}_t \cdot \underline{r})} (1 + \frac{\sigma}{s\epsilon}) \Big[$$

(3.7)

$$\frac{1}{a} \hat{g}_1(\frac{s}{a}, \xi_1, \xi_2, 2\cos^2\varphi_t, z/\delta) - \nu \frac{1}{a} \hat{g}_0(\frac{s}{a}, \xi_1, \xi_2, z/\delta)\Big],$$

with $\xi_1 = -s_1/a$, and $\xi_2 = -s_2/a$, and where δ denotes the skin depth for the high-frequency/low-loss approximation (Klaasen [1]), given by

(3.8) $$\delta = 2 \epsilon v \cos \varphi_t / \sigma.$$

Eq.(3.7) denotes a wave propagating in the direction of $\underline{\chi}_t$, which is given by

(3.9) $$\underline{\chi}_t = \sin \varphi_t \underline{i}_x + \cos \varphi_t \underline{i}_z.$$

With the help of the identities

$$\text{(3.10)}\qquad \lim_{\nu\to 1} (1-\nu)\, s_1 = -\, a \cos 2\varphi_t, \qquad \lim_{\nu\to 1} s_2 = -\,\frac{\sigma}{\epsilon} \cos^2\varphi_t \,/\, \cos 2\varphi_t,$$

it can be proven that eq.(3.7) is also valid if the angle of incidence equals the Brewster angle.
After decomposition of $\hat{g}_0$ and $\hat{g}_1$ in partial parts we obtain for $\hat{g}_0$

$$\text{(3.11)}\qquad \hat{g}_0 = \frac{1}{\xi_1 - \xi_2}\left[\frac{\xi_1 - 2}{s+\xi_1} - \frac{\xi_2 - 2}{s+\xi_2}\right] \frac{1}{\sqrt{(s+1)^2-1}}\, e^{-z\left(\sqrt{(s+1)^2-1}\;-(s+1)\right)},$$

and the expression for $\hat{g}_1$ is rewritten as

$$\text{(3.12)}\qquad \hat{g}_1 = \frac{1}{\xi_1 - \xi_2}\left[\frac{\xi_1 - \lambda}{s+\xi_1} - \frac{\xi_2 - \lambda}{s+\xi_2}\right] e^{-z\left(\sqrt{(s+1)^2-1}\;-(s+1)\right)}.$$

The expressions for $\hat{g}_0$ and $\hat{g}_1$ are ready to be transformed to the time domain, using the standard inverse Laplace transforms listed in Appendix B. After doing so, we obtain for $g_0(t)$

$$\text{(3.13)}\qquad g_0(t,\xi_1,\xi_2,z) = \frac{1}{\xi_1 - \xi_2}\left\{(\xi_1 - 2)e^{-\xi_1 t} - (\xi_2 - 2)e^{-\xi_2 t}\right\} * e^{-t}\left\{(\partial_t - 1) I_0(\sqrt{t^2 + 2tz}) + \delta(t)\right\},$$

with $t \geq 0$, and where $I_0(t)$ denotes the modified Bessel function of the

first kind and order zero. The asterisk in eq.(3.13) denotes the convolution operator, which is given by

$$g(t) = h(t) * f(t) = \int_0^t h(t-\tau)\, f(\tau)\, d\tau. \tag{3.14}$$

The time-domain expression for $g_1(t)$ is given by

$$\begin{aligned} g_1(t,\xi_1,\xi_2,\lambda,z) = \frac{1}{\xi_1 - \xi_2} &\left\{(\xi_1 - \lambda)e^{-\xi_1 t} - (\xi_2 - \lambda)e^{-\xi_2 t}\right\} * \\ &e^{-t}\left\{ z\,\frac{I_1(\sqrt{t^2 + 2tz})}{\sqrt{t^2 + 2tz}} + \delta(t)\right\}, \end{aligned} \tag{3.15}$$

again with $t \geq 0$, and where $I_1(t)$ denotes the modified Bessel function of the first kind and order one.
After introduction of the auxiliary functions h_0, h_1 and h_2 defined by

$$\begin{aligned} h_0(t,\beta,z) &= e^{-\beta t} \int_0^t e^{-(1-\beta)\tau}\, I_0(\sqrt{\tau^2 + 2\tau z})\, d\tau, \\ h_1(t,\beta,z) &= e^{-\beta t}\, [\, 1 + z \int_0^t e^{-(1-\beta)\tau}\, \frac{I_1(\sqrt{\tau^2 + 2\tau z})}{\sqrt{\tau^2 + 2\tau z}}\, d\tau\,], \\ h_2(t,z) &= e^{-t}\, I_0(\sqrt{t^2 + 2tz}), \end{aligned} \tag{3.16}$$

and after applying the convolution operator, the expressions for g_0 and

g_1 are expressed as

(3.17) $$g_0 = h_2 + \frac{1}{\xi_1 - \xi_2}\Big[\xi_2(\xi_2-2)h_0(t,\xi_2,z) - \xi_1(\xi_1-2)h_0(t,\xi_1,z)\Big],$$

(3.18) $$g_1 = \frac{1}{\xi_1 - \xi_2}\Big[(\xi_1-\lambda)h_1(t,\xi_1,z) - (\xi_2-\lambda)h_1(t,\xi_2,z)\Big].$$

The functions g_0 and g_1 are only causal for $\xi_{1,2} > 0$. Now ξ_2 is always positive, but ξ_1 can become negative if :

- $\epsilon_r \geq \mu_r$, and
- $\varphi_i \geq \varphi_b^H$.

Therefore, the presented solution is only causal for angles of incidence smaller than the Brewster angle φ_b^H.
Figs. 3.1-3.3 show the functions h_0, h_1 and h_2 as a function of time for different β.
After applying the inverse Laplace transform to eq.(3.7), the transmitted magnetic field in the time domain is given by

(3.19) $$H_y^t(\underline{r},t) = \frac{2}{1-\nu^2}\, U(t')\, e^{-z'}\Big(1 + \frac{\sigma}{\epsilon}\int_0^{t'} d\tau\Big)\Big[g_1(at',\xi_1,\xi_2,2\cos^2\varphi_t,z') - \nu g_0(at',\xi_1,\xi_2,z')\Big],$$

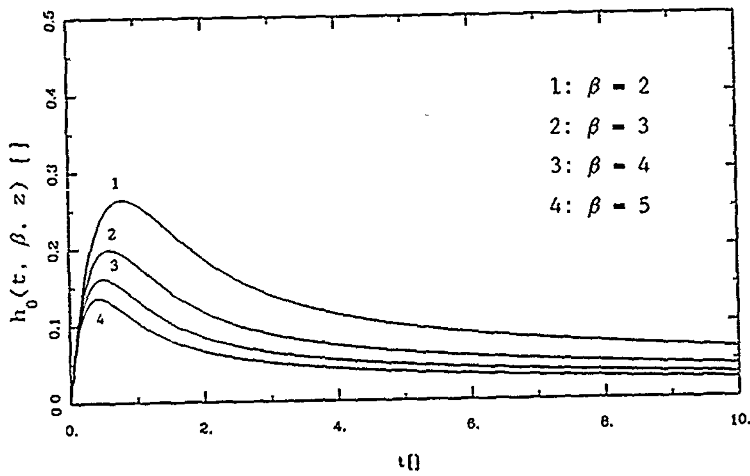
where t' denotes the retarded time given by

(3.20) $$t' = t - (\underline{\chi}_t\cdot\underline{r})/v,$$

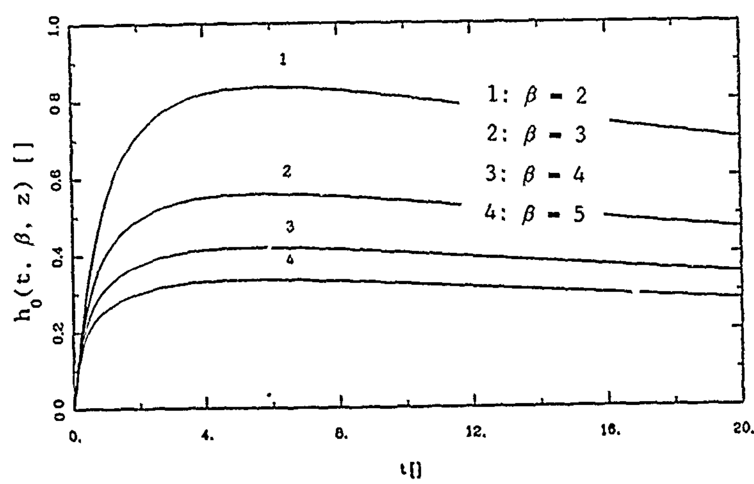
and with

(3.21) $$z' = z\,/\,\delta.$$

In obtaining eq.(3.19) we have also used the property $f(at)=\mathscr{L}^{-1}\{\frac{1}{a}\hat{f}(\frac{s}{a})\}$.



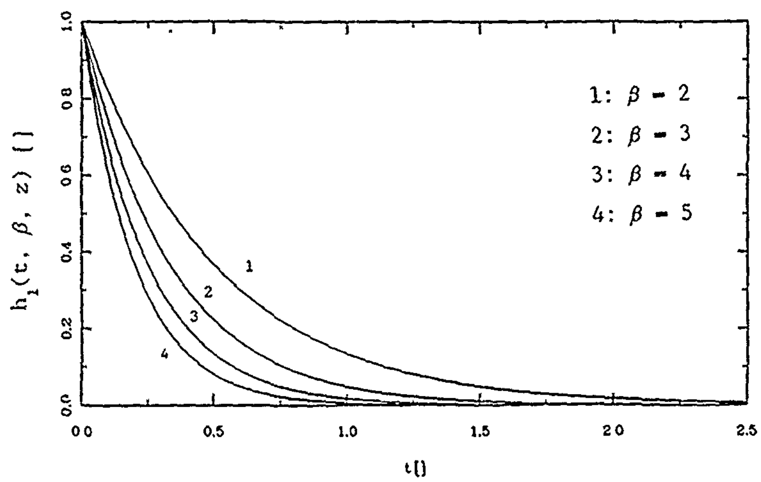
(a)



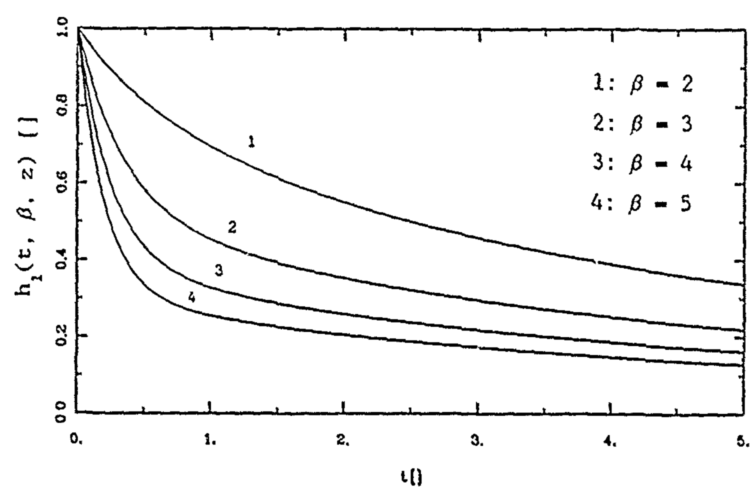
(b)

Fig. 3.1. The function $h_0(t, \beta, z)$ as a function of time

a) $z = 0$,
b) $z = 3$.



(a)



(b)

Fig. 3.2. The function $h_1(t, \beta, z)$ as a function of time
a) z = 0,
b) z = 3.

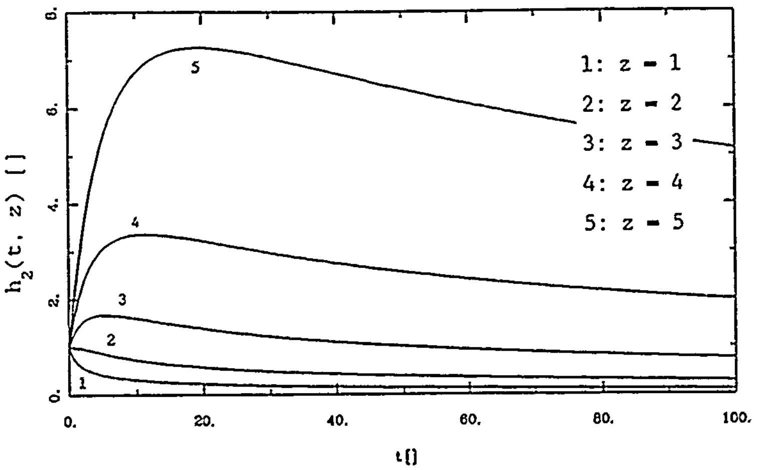


Fig. 3.3. The function $h_2(t, z)$ as a function of time.

Since

$$\frac{\sigma}{\epsilon}\int_0^{t'} g_0(a\tau)\,d\tau = \frac{2\cos^2\varphi_t}{\xi_1 - \xi_2}\Big[\xi_1(\xi_1-2)\,h_0(at',\xi_1,z') - \xi_2(\xi_2-2)\,h_0(at',\xi_2,z')\Big], \tag{3.22}$$

and

$$\frac{\sigma}{\epsilon}\int_0^{t'} g_1(a\tau)\,d\tau = (1-\nu^2)\,h_1(at',0,z') + 2\cos^2\varphi_t\,\frac{1}{\xi_1\xi_2}\cdot\frac{1}{\xi_1-\xi_2}\Big[\xi_1(\xi_2-2\cos^2\varphi_t)h_1(at',\xi_2,z') - \xi_2(\xi_1-2\cos^2\varphi_t)h_1(at',\xi_1,z')\Big], \tag{3.23}$$

we finally obtain for the transmitted magnetic field in $\mathcal{D}_2$

$$H_y^t(\underline{r},t) = 2\,U(t')\,e^{-z'}\Bigg\{h_1(at',0,z') - \frac{\nu}{1-\nu^2}h_2(at',z') + \frac{1}{1-\nu^2}\cdot\frac{1}{\xi_1-\xi_2}\Bigg[\frac{(\xi_1-2\cos^2\varphi_t)^2}{\xi_1}h_1(at',\xi_1,z') - \frac{(\xi_2-2\cos^2\varphi_t)^2}{\xi_2}h_1(at',\xi_2,z') + \nu(\xi_1-2)(\xi_1-2\cos^2\varphi_t)\,h_0(at',\xi_1,z') - \nu(\xi_2-2)(\xi_2-2\cos^2\varphi_t)\,h_0(at',\xi_2,z')\Bigg]\Bigg\}, \tag{3.24}$$

The transmitted wave is, of course, not a plane wave, but we can see from the expression for the retarded time t' given by eq.(3.20) together with eq.(3.24), that the wave front propagates with speed v.

3.2 Transmitted electric fields

The unit-step response of the transmitted electric fields in $\mathcal{D}_2$ follows from eq.(2.18). Using the same notation as in the previous section, we write for the transmitted electric fields in $\mathcal{D}_2$

$$\hat{E}_x^t(\underline{r},s) = 2\, Z \cos\varphi_t\, s^{-1} \frac{\sqrt{s^2+2as}}{s+\frac{\sigma}{\epsilon}+\nu\sqrt{s^2+2as}} \times e^{-s\frac{x}{v}\sin\varphi_t - \frac{z}{v}\cos\varphi_t\sqrt{s^2+2as}},$$

$$\hat{E}_z^t(\underline{r},s) = -2\, Z \sin\varphi_t \frac{1}{s+\frac{\sigma}{\epsilon}+\nu\sqrt{s^2+2as}} \times e^{-s\frac{x}{v}\sin\varphi_t - \frac{z}{v}\cos\varphi_t\sqrt{s^2+2as}}, \tag{3.25}$$

where Z denotes the dielectric wave impedance of $\mathcal{D}_2$ given by

$$Z = \sqrt{\mu/\epsilon}. \tag{3.26}$$

After multiplying the denominator as well as the numerator by $(s+\frac{\sigma}{\epsilon}) - \nu\sqrt{s^2+2as}$, and using the functions $\hat{g}_0$ and $\hat{g}_1$ defined by

eq.(3.6), this can be written as

$$\hat{E}_x^t = \frac{2}{1-\nu^2} Z \cos\varphi_t \, e^{-z'} e^{-\frac{s}{v}(\underline{\chi}_t \cdot \underline{r})} \Big[(1 + \frac{\sigma}{s\epsilon}) \frac{1}{a} \hat{g}_0(\frac{s}{a},\xi_1,\xi_2,z') - \nu \frac{1}{a} \hat{g}_1(\frac{s}{a},\xi_1,\xi_2,2,z') \Big],$$

(3.27)

$$\hat{E}_z^t = -\frac{2}{1-\nu^2} Z \sin\varphi_t \, e^{-z'} \, e^{-\frac{s}{v}(\underline{\chi}_t \cdot \underline{r})} \Big[\frac{1}{a} \hat{g}_1(\frac{s}{a},\xi_1,\xi_2,2\cos^2\varphi_t,z') - \nu \frac{1}{a} \hat{g}_0(\frac{s}{a},\xi_1,\xi_2,z') \Big] .$$

The parameters ξ_1, ξ_2 are again given by $\xi_1 = -s_1/a$, and $\xi_2 = -s_2/a$. The corresponding time-domain expressions are then found to be

$$E_x^t(\underline{r},t) = \frac{2}{1-\nu^2} Z \cos\varphi_t \, U(t') \, e^{-z'} \Big\{ + (1 + \frac{\sigma}{\epsilon} \int_0^{t'} d\tau) \, g_0(at',\xi_1,\xi_2,z')$$

(3.28)

$$- \nu \, g_1(at',\xi_1,\xi_2,2,z') \Big\}$$

$$E_z^t(\underline{r},t) = -\frac{2}{1-\nu^2} Z \sin\varphi_t \, U(t') \, e^{-z'} \Big\{ g_1(at',\xi_1,\xi_2,2\cos^2\varphi_t,z') - \nu g_0(at',\xi_1,\xi_2,z') \Big\}.$$

3.3 Reflected fields

Because the propagation factor of the reflected fields denotes simply a plane wave propagating in the direction of $\underline{\chi}_r$ given by

(3.29) $$\underline{\chi}_r = \sin\varphi_i \underline{i}_x - \cos\varphi_i \underline{i}_z,$$

the reflected waves can be found easily from the transmitted fields at the boundary, since

(3.30) $$\hat{R}^H = \hat{T}^H - 1.$$

Using this boundary condition, we write directly in the time domain for the reflected magnetic field

(3.31) $$H_y^r(\underline{r},t) = H_y^t(z=0,t-(\underline{\chi}_r\cdot\underline{r})/c_0) - U(t-(\underline{\chi}_r\cdot\underline{r})/c_0),$$

which is a plane wave propagating in the direction of $\underline{\chi}_r$.
The reflected electric fields follow simply from eq.(2.22), and are found to be

(3.32) $$E_x^r(\underline{r},t) = -\cos\varphi_i\, Z_0\, H_y^r(\underline{r},t),$$
$$E_z^r(\underline{r},t) = -\sin\varphi_i\, Z_0\, H_y^r(\underline{r},t).$$

4 TIME-DOMAIN SOLUTION FOR A HORIZONTALLY POLARIZED INCIDENT FIELD WITH A UNIT-STEP WAVEFORM

To obtain expressions in the time domain for a horizontally polarized incident field, the same procedure is used in this chapter as in Chapter 3. This means that we determine the unit-step response, which is obtained from the Laplace-domain expressions by applying the inverse Laplace transform analytically.

4.1 Transmitted electric field

The transmitted field for horizontal polarization is given by eq.(2.27). Since we determine the unit-step response, we set $\hat{E}_0$ equal to s^{-1}. The transmitted electric field can then be expressed as

$$(4.1) \qquad \hat{E}_y^t = \frac{2}{s + \rho\sqrt{s^2 + 2as}} \, e^{-s\frac{x}{v}\sin\varphi_t - \frac{z}{v}\cos\varphi_t\sqrt{s^2+2as}} ,$$

with ρ given by eq.(2.31). Notice that ρ can be compared with ν in Chapter 3. If $\epsilon_r > \mu_r$, ρ is always larger than 1. If $\epsilon_r \le \mu_r$ a similar effect occurs as with the case of vertical polarization. For angles of incidence larger than or equal to the Brewster angle for horizontal polarization, ρ can become smaller than or equal to one. The Brewster angle for horizontal polarization, denoted by φ_b^E, can be obtained from

$$(4.2) \qquad \tan\varphi_b^E = \left[\frac{\mu_r(\mu_r - \epsilon_r)}{\epsilon_r\mu_r - 1}\right]^{1/2} . \qquad \epsilon_r \le \mu_r$$

After multiplying the denominator as well as the numerator of eq.(4.1)

by $\rho\sqrt{s^2 + 2as} - s$, we obtain

(4.3) $$\hat{E}_y^t = \frac{2s^{-1}}{\rho^2-1} \frac{\rho\sqrt{s^2 + 2as} - s}{s + a\dfrac{2\rho^2}{\rho^2-1}} e^{-(s\frac{x}{v}\sin\varphi_t + \frac{z}{v}\cos\varphi_t\sqrt{s^2 + 2as})}.$$

From eq.(4.3), we observe that the transmitted electric field has a pole at $s = 0$, and a pole for $\rho > 1$, given by

(4.4) $$s = -a\frac{2\rho^2}{\rho^2-1}. \qquad \rho > 1$$

If $\rho \le 1$ the transmission coefficient does not have a pole, since in this case eq.(4.4) represents a zero of the reflection coefficient. Using functions similar to $\hat{g}_0$ and $\hat{g}_1$ as used in the previous chapter, the expressions for the transmitted electric field can be rewritten as

(4.5) $$\hat{E}_y^t = \frac{2}{\rho^2-1} e^{-z'} e^{-\frac{s}{v}(\underline{\chi}_t \cdot \underline{r})} \left(\rho\frac{1}{a}\hat{f}_0(\frac{s}{a}, \xi, z') - \frac{1}{a}\hat{f}_1(\frac{s}{a}, \xi, z')\right),$$

with $z' = z/\delta$, $\xi = \dfrac{2\rho^2}{\rho^2-1}$ and with

(4.6)
$$\hat{f}_0(s, \xi, z) = (1 + \frac{2 - \xi}{s + \xi}) \frac{e^{-z\left(\sqrt{(s+1)^2 - 1} - (s+1)\right)}}{\sqrt{(s+1)^2 - 1}},$$

$$\hat{f}_1(s, \xi, z) = \frac{1}{s + \xi} e^{-z\left(\sqrt{(s+1)^2 - 1} - (s+1)\right)}.$$

In eq.(4.5), $\underline{\chi}_t$ denotes the direction of propagation given by

(4.7) $$\underline{\chi}_t = \sin \varphi_t \underline{i}_x + \cos \varphi_t \underline{i}_z .$$

After applying the inverse Laplace transform with the help of the standard Laplace transforms listed in Appendix B, we obtain for f_0 and f_1

(4.8)
$$f_0(t,\xi,z) = \left\{ \delta(t) + (2 - \xi)\, e^{-\xi t} \right\} * e^{-t} I_0(\sqrt{t^2 + 2tz}),$$

$$f_1(t,\xi,z) = e^{-\xi t} * e^{-t} \left\{ z\, \frac{I_1(\sqrt{t^2 + 2tz})}{\sqrt{t^2 + 2tz}} + \delta(t) \right\},$$

for $t \geq 0$, and where we have used the property $\mathcal{L}^{-1}\{\hat{f}(s+1)\} = e^{-t} f(t)$. Using the auxiliary functions h_0, h_1, and h_2 given by eq.(3.16) the expressions for f_0 and f_1 are expressed as

(4.9)
$$f_0(t,\xi,z) = h_2(t,z) + (2 - \xi) h_0(t,\xi,z),$$

$$f_1(t,\xi,z) = h_1(t,\xi,z).$$

The functions f_0 and f_1 are only causal for $\xi > 0$. Therefore the solutions for f_0 and f_1 are only valid provided that $\xi > 0$. Remember that $\xi < 0$ if :

- $\epsilon_r \leq \mu_r$, and
- $\varphi_i \geq \varphi_b^E$.

In practical situations $\mu_r = 1$, so that this situation is not met in practice for horizontal polarization.

After applying the inverse Laplace transform to eq.(4.5) analytically,

the time-domain expression for the transmitted electric field is given by

$$E_y^t(\underline{r},t) = \frac{2}{\rho^2-1}\, U(t')\, e^{-z'}\left\{ \rho f_0(at',\xi,z') - f_1(at',\xi,z')\right\}, \tag{4.10}$$

where t' denotes the retarded time given by

$$t' = t - (\underline{\chi}_t \cdot \underline{r})/v. \tag{4.11}$$

Hence, the transmitted electric field is attenuated by a factor $e^{-z/\delta}$ while propagating in $\mathcal{D}_2$, and is dependent on at' and z' only. Of course, the transmitted electric field is not a uniform plane wave, but from the retarded time one can observe that the wave front travels with speed v.

4.2 Transmitted magnetic fields

The transmitted magnetic fields in $\mathcal{D}_2$ follow from eq.(2.27). The normal transmitted magnetic field is simply found to be

$$H_z^t(\underline{r},t) = Y \sin\varphi_t\, E_y^t(\underline{r},t), \tag{4.12}$$

where Y denotes the dielectric wave admittance of $\mathcal{D}_2$ given by

$$Y = \sqrt{\epsilon/\mu}. \tag{4.13}$$

Using the same notation as in section 4.1, we write for the

transverse transmitted magnetic field in $\mathcal{D}_2$

$$\hat{H}_x^t(\underline{r},s) = -2\,Y\cos\varphi_t\,s^{-1}\,\frac{\sqrt{s^2+2as}}{s+\rho\sqrt{s^2+2as}}\times \tag{4.14}$$

$$e^{-s\frac{x}{v}\sin\varphi_t - \frac{z}{v}\cos\varphi_t\sqrt{s^2+2as}}.$$

After multiplying the denominator as well as the numerator of eq.(4.14) by $\rho\sqrt{s^2+2as} - s$, and using the functions $\hat{f}_0$ and $\hat{f}_1$ defined by eq.(4.6), eq.(4.14) can be rewritten as

$$\hat{H}_x^t = -\frac{2}{\rho^2-1}\,Y\cos\varphi_t\,e^{-z'}e^{-\frac{s}{v}(\underline{\gamma}_t\cdot\underline{r})}\Big[\tag{4.15}$$

$$\rho\left(1+\frac{2a}{s}\right)\frac{1}{a}\,\hat{f}_1\left(\frac{s}{a},\xi,z'\right) - \frac{1}{a}\,\hat{f}_0\left(\frac{s}{a},\xi,z'\right)\Big].$$

This yields for the transverse magnetic field in the time domain

$$H_x^t(\underline{r},t) = -\frac{2}{\rho^2-1}\,Y\cos\varphi_t\,U(t')\,e^{-z'}\Big\{ \tag{4.16}$$

$$\rho\left(1+2a\int_0^{t'}d\tau\right)f_1(at',\xi,z') - f_0(at',\xi,z')\Big\}.$$

Since

$$2a\int_0^{t'} f_1(a\tau,\lambda,z')d\tau = \frac{2}{\lambda}\left(f_1(at',0,z') - f_1(at',\lambda,z')\right), \tag{4.17}$$

we finally obtain for the transverse magnetic field in the time domain

$$H_x^t(\underline{r},t) = -\frac{2}{\rho^2-1} Y \cos\varphi_t\, U(t')\, e^{-z'} \Big\{$$

$$\Big[f_1(at',\xi,z') + (\rho^2-1)\, f_1(at',0,z') \Big] / \rho \qquad (4.18)$$

$$- f_0(at',\xi,z') \Big\}.$$

4.3 Reflected fields

The reflected fields denote a plane wave propagating in the direction of $\underline{\chi}_r$ given by

$$\underline{\chi}_r = \sin\varphi_i \underline{i}_x - \cos\varphi_i \underline{i}_z. \qquad (4.19)$$

Using the same reasoning as in section 3.3, the reflected electric field in the time domain is written as

$$E_y^r(\underline{r},t) = E_y^t(z=0,t-(\underline{\chi}_r\cdot\underline{r})/c_0) - U(t-(\underline{\chi}_r\cdot\underline{r})/c_0). \qquad (4.20)$$

The reflected magnetic fields follow simply from eq.(2.28), and are found to be

$$\begin{aligned} H_x^r(\underline{r},t) &= \cos\varphi_i\, Y_0\, E_y^r(\underline{r},t), \\ H_z^r(\underline{r},t) &= \sin\varphi_i\, Y_0\, E_y^r(\underline{r},t). \end{aligned} \qquad (4.21)$$

5 NUMERICAL RESULTS

In this Chapter numerical results are presented for the scattering of a unit-step and a Nuclear ElectroMagnetic Pulse (NEMP) by a plane interface. For simplicity, only results for horizontally polarized incident waves are presented.
In Section 5.1, some aspects about the numerical implementation are discussed.

5.1 Numerical implementation

The expressions derived in the previous chapters are ready to be implemented in a computer program except for the functions h_0 and h_1 given by eq.(3.16). The most efficient implementation of the time-domain expressions uses a time-marching procedure, i.e. the fields at time instant $t+\Delta t$ are found from the fields at the time instant t, where Δt is the time step of the time-marching procedure. Consequently, we need to know h_0 and h_1 at the time instant $t+\Delta t$ in terms of h_0 and h_1, respectively, at the time instant t. After some simple manipulation, we arrive at

$$h_0(t+\Delta t,\beta,z) = e^{-\beta\Delta t}\Big(h_0(t,\beta,z) + e^{-\beta t}\int_t^{t+\Delta t} e^{-(1-\beta)\tau}\, I_0(\sqrt{\tau^2+2\tau z})\, d\tau\Big),$$

$$h_1(t+\Delta t,\beta,z) = e^{-\beta\Delta t}\Big(h_1(t,\beta,z) + z\, e^{-\beta t}\int_t^{t+\Delta t} e^{-(1-\beta)\tau}\, \frac{I_1(\sqrt{\tau^2+2\tau z})}{\sqrt{\tau^2+2\tau z}}\, d\tau\Big). \tag{5.1}$$

During the numerical experiments, of which the next sections present the results, the integrals over the interval [t, t+Δt] were computed with an adaptive trapezoidal rule, such that the relative error in the total field solution due to the errors in h_0 and h_1 was always less than 1%, independently on the time step Δt.

5.2 Unit-step incident field

Let the incident field be horizontally polarized with a unit-step waveform. Then the transmitted electric field is given by eq.(4.10) and the reflected electric field is given by eq.(4.20). The value of the fields at $t = 0$ can be found from Abel's theorem, which states that

$$(5.2) \qquad \lim_{s\to\infty} s\hat{f}(s) = f(0).$$

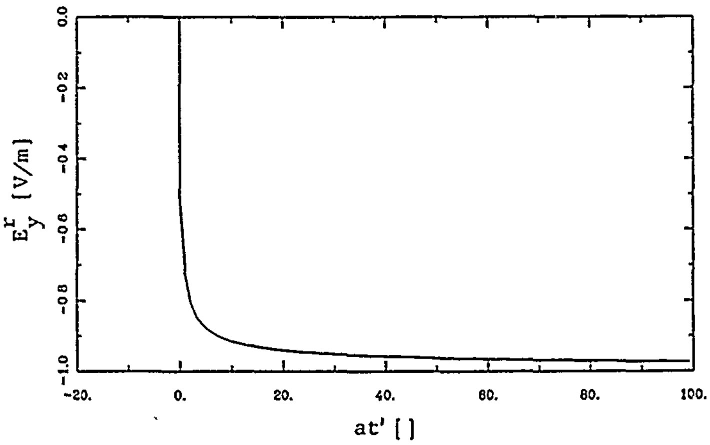
Using Abel's theorem, we obtain from the s-domain, cf. eq.(4.1),

$$(5.3) \qquad \lim_{t\to 0} E_y^t = \frac{2}{1+\rho},$$

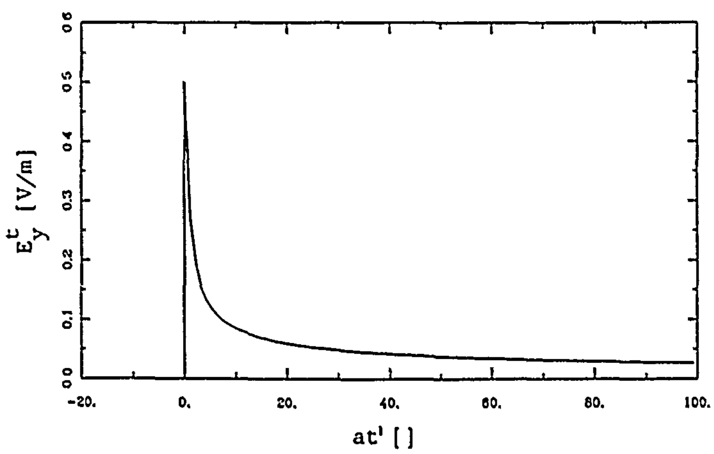
and consequently,

$$(5.4) \qquad \lim_{t\to 0} E_y^r = \frac{1-\rho}{1+\rho}.$$

This result agrees with the time-domain equations (4.10) and (4.20). Fig. 5.1 shows some results, which were obtained from the time-domain expressions, with $\epsilon_r = 9$, $\mu_r = 1$, $\sigma = 1 \times 10^{-3}$ and the angle of incidence $\varphi_i = 0$. Consequently, $E_y^t(\underline{r}, 0) = 1/2$ and $E_y^r(\underline{r}, 0) = -1/2$.



(a)



(b)

Fig. 5.1 Electric field as a function of at'. $\epsilon_r = 9$, $\sigma = 1 \times 10^{-3}$, $\mu_r = 1$, $\varphi_i = 0$ and $z = 0$.
a) reflected field,
b) transmitted field.

From eq.(4.20) one can observe that the reflected fields depend on at' and ρ only. Therefore, if ρ is constant and we vary the conductivity σ, and thereby at', only the time scale changes. Consequently, the rise time and half-width of the reflected fields vary inversely proportional with σ. Fig. 5.2 shows the reflected electric field for various ρ but with fixed σ.

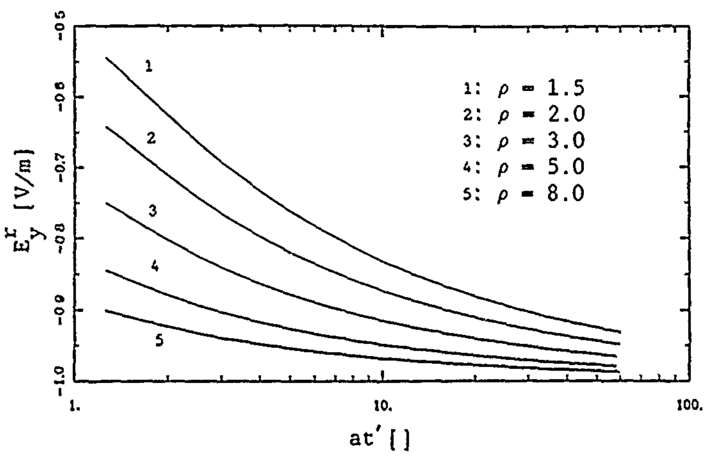


Fig. 5.2 Reflected electric field as a function of at' (logarithmic scale for clarity) and ρ. Conductivity $\sigma = 1 \times 10^{-3}$, $\varphi_i = 0$.

The rise time for any other conductivity σ can be easily found from this figure in the following way. Select the curve with the correct ρ. Read the normalized rise time t_r' of that curve. The rise time for any other conductivity σ is then determined from

(5.5) $$t_r = 2 \rho^2 t_r' / \sigma.$$

The transmitted fields also depend on z/δ, so a similar observation

cannot be made. Fig. 5.3 shows the transmitted fields for $\epsilon_r = 9$ and for different ratios of z/δ.

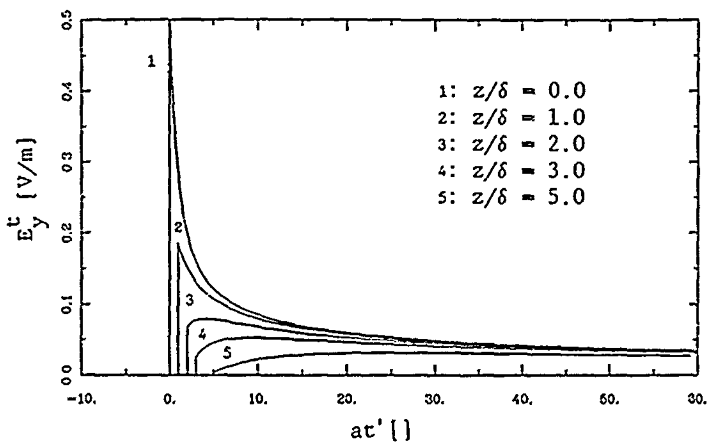


Fig. 5.3 Transmitted electric field as a function of at' for different ratios of z/δ. $\epsilon_r = 9$, $\mu_r = 1$, $\sigma = 1 \times 10^{-3}$, $\varphi_i = 0$.

For late time the transmitted fields are determined mainly by h_2. Therefore,

$$(5.6) \qquad E_y^t \approx \frac{2\rho}{\rho^2-1} e^{-z'} h_2(t', z') \approx \frac{2\rho}{\rho^2-1} e^{-(t'+z')} I_0(t'+z'),$$

where we have used the abbreviations $z' = z/\delta$ and $t' = a(t-(\underline{x}_t \cdot \underline{r})/v)$, and that $t' \gg z'$. The term $e^{-(t'+z')} I_0(t'+z')$ can be rewritten as (see Abramowitz [6]), with $t' \gg z'$,

$$(5.7) \qquad e^{-(t'+z')} I_0(t'+z') \approx 0.399 / \sqrt{t'},$$

which is independent on z'. Therefore, for late time the transmitted fields are proportional to $1/\sqrt{t'}$, which is also shown by Fig.5.3.

5.3 NEMP incident field

In this section results are presented for an incident Nuclear ElectroMagnetic Pulse (NEMP). The NEMP is a transient signal with a very-short rise time of about 5 nsec and a large electric field strength with a peak value of 50 kV/m. The NEMP is approximated by a double exponential function given by (Bell laboratory waveform)

$$E_0(t) = A\,(e^{-\alpha t} - e^{-\beta t}), \tag{5.8}$$

with

$$\begin{aligned} A &= 5.278 \times 10^4 \ [\tfrac{V}{m}], \\ \alpha &= 3.705 \times 10^6 \ [s^{-1}], \\ \beta &= 3.908 \times 10^8 \ [s^{-1}]. \end{aligned} \tag{5.9}$$

The transmitted electric field due to a NEMP is obtained from the unit-step response in the way described in Appendix A. We then find from eqs.(5.8), (4.10) and (A.6)

$$E_y^t(\underline{r}, t) = A\Big(F(at', \beta/a, z') - F(at', \alpha/a, z')\Big), \tag{5.10}$$

where

$$\begin{aligned} t' &= t - (\underline{\chi}_t \cdot \underline{r})/v, \\ z' &= z/\delta, \end{aligned} \tag{5.11}$$

and where F is given by

$$F(t,\ \alpha,\ z) = \frac{2\alpha}{\rho^2-1}\, e^{-z} \int_0^t e^{-\alpha(t-\tau)} \left\{ \rho f_0(\tau,\ \xi,\ z) - f_1(\tau,\ \xi,\ z)\right\} d\tau. \tag{5.12}$$

In eq.(5.12), ξ denotes the normalized pole of the transmission coefficient, which is given by

$$\xi = \frac{2\rho^2}{\rho^2-1}. \tag{5.13}$$

Now using

$$\begin{aligned}
&\int_0^t e^{-\alpha(t-\tau)}\, h_0(\tau,\ \xi,\ z)\, d\tau = \frac{1}{\alpha-\xi}\,(h_0(t,\xi,z) - h_0(t,\alpha,z)),\\
&\int_0^t e^{-\alpha(t-\tau)}\, h_1(\tau,\ \xi,\ z)\, d\tau = \frac{1}{\alpha-\xi}\,(h_1(t,\xi,z) - h_1(t,\alpha,z)),\\
&\int_0^t e^{-\alpha(t-\tau)}\, h_2(\tau,\ z)\, d\tau = h_0(t,\alpha,z),
\end{aligned} \tag{5.14}$$

eq.(5.12) yields together with eq.(4.9)

$$F(t,\ \alpha,\ z) = \frac{2}{\rho^2-1}\,\frac{\alpha}{\alpha-\xi}\, e^{-z} \Big[\rho\big[\ (\alpha-2)\ h_0(t,\alpha,z) - (\xi-2)\ h_0(t,\xi,z)\ \big] \tag{5.15}$$

$$+ h_1(t,\alpha,z) - h_1(t,\xi,z) \Big].$$

Obviously, the reflected electric field is then given by

$$E_y^r(\underline{r}, t) = A\Big(F(at', \beta/a, 0) + e^{-\beta t'} - F(at', \alpha/a, 0) - e^{-\alpha t'}\Big), \tag{5.16}$$

where the retarded time t' is now given by

$$t' = t-(\underline{\chi}_r \cdot \underline{r})/c_0. \tag{5.17}$$

Figs.5.4 and 5.5 show the reflected and transmitted NEMP, respectively, for $\epsilon_r = 9$, $\mu_r = 1$, $\varphi_i = 0$.

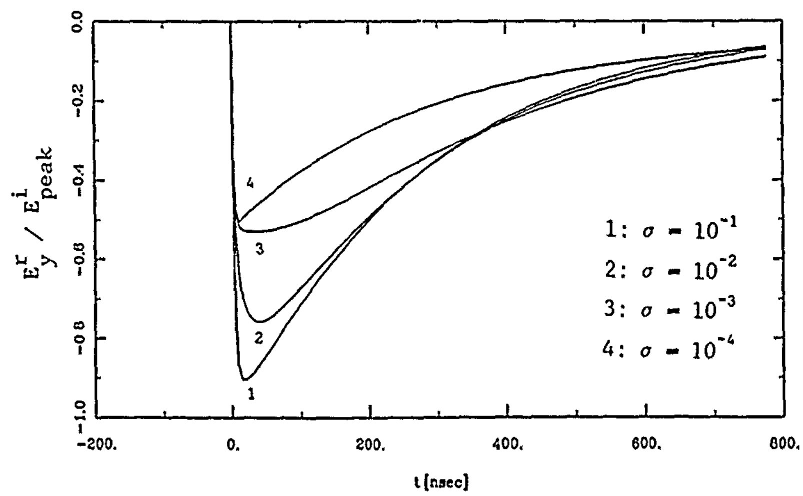
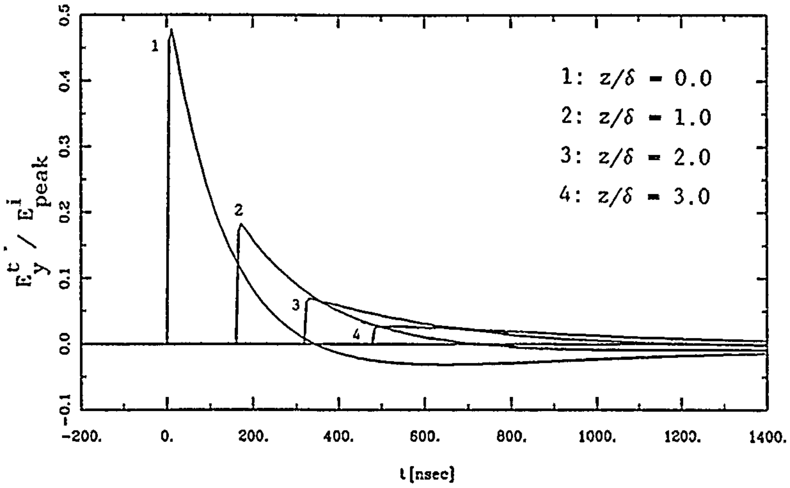
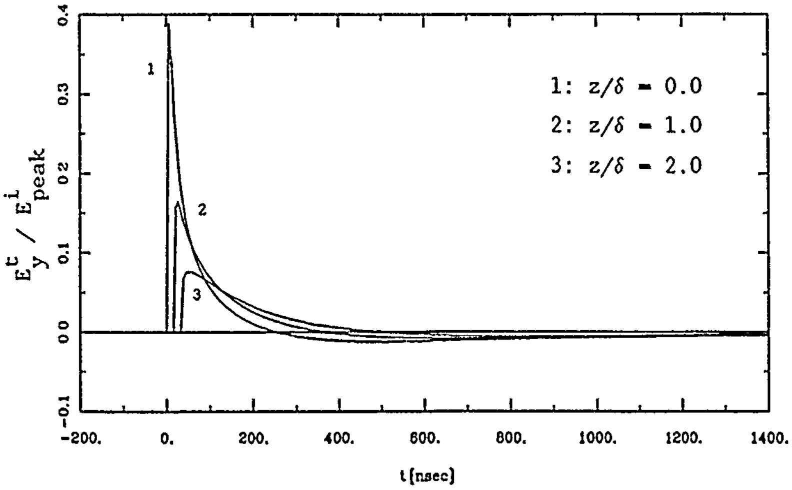


Fig. 5.4. Reflected NEMP for various conductivity. $\epsilon_r = 9$, $\mu_r = 1$, $\varphi_i = 0$, $z = 0$.



(a)



(b)

Fig. 5.5. Transmitted NEMP for various z. $\epsilon_r = 9$, $\mu_r = 1$, $\varphi_i = 0$.
a) $\sigma = 1\times10^{-3}$,
b) $\sigma = 1\times10^{-2}$.

6 CONCLUSIONS

It was shown that time-domain expressions for the scattered electromagnetic fields due to an incident field with unit-step waveform, can be obtained from the Laplace-domain expressions by an analytical inverse Laplace transform. For both vertical and horizontal polarization exact time-domain expressions were obtained. Therefore, the discussion going on in IEEE Transactions on Electromagnetic Compatibility, Harmuth [10]-[11], about the necessity of modifying Maxwell's equation is hereby settled.

The scattered fields due to an incident field with any other waveform than the unit-step, can be obtained from the derived unit-step responses. In Appendix A a method is presented how to do this. This method typically involves a kind of convolution integral.
In those cases where the waveform of the incident field $f(t)$ has the property $f(t-\tau) = f_1(t) \cdot f_2(\tau)$, the convolution can be determined very efficiently by a time-marching procedure. This is because the part $f_1(t)$ can be taken outside the integral sign so that the integrand becomes time independent. This property has been demonstrated by applying the derived theory to an incident field with a NEMP waveform.

The derived expressions for vertical polarization are valid only for angles of incidence smaller than the Brewster angle. For horizontal polarization this situation is not met in practice. It is expected that in the near future correct solutions will be found for angles of incidence larger than the Brewster angle.

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RESPONSE TO OTHER INCIDENT WAVEFORMS FROM THE UNIT-STEP RESPONSE

The impulse response of a linear system is the response of that system to a delta function excitation. Let the impulse response of such a system be denoted by h(t). Then the response to an arbitrary input f(t) is given by

$$\text{(A.1)} \qquad g(t) = \int_0^t h(t-\tau)\ f(\tau)\ d\tau = \int_0^t h(\tau)\ f(t-\tau)\ d\tau,$$

where we have assumed that the system is causal, i.e. $h(t) = 0$ for $t < 0$, and that $f(t) = 0$ for $t < 0$.
For the purpose of the derivation we rewrite eq.(A.1) as follows

$$\text{(A.2)} \qquad g(t) = \int_{-\infty}^{\infty} h(\tau)\ U(\tau)\ f(t-\tau)\ U(t-\tau)\ d\tau,$$

where U(t) denotes the unit-step function given by

$$\text{(A.3)} \qquad U(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

Now, let w(t) denote the response to a unit-step function. Then w(t) can be obtained from the impulse response by

$$\text{(A.4)} \qquad w(t) = \int_{-\infty}^{\infty} h(\tau)\ U(\tau)\ U(t-\tau)\ d\tau = \int_{-\infty}^{t} h(\tau)\ U(\tau)\ d\tau.$$

Consequently, observe that $\partial_t w(t) = h(t)\ U(t)$. Therefore, eq.(A.2) can

be rewritten as

$$(A.5) \qquad g(t) = \int_{-\infty}^{\infty} \partial_\tau w(\tau)\ f(t-\tau)\ U(t-\tau)\ d\tau,$$

which yields after partial integration

$$(A.6) \qquad \begin{aligned} g(t) &= \int_{-\infty}^{\infty} w(\tau)\ \partial_t(f(t-\tau)\ U(t-\tau))\ d\tau = \\ &w(t)\ f(0) + \int_{0}^{t} w(\tau)\ f'(t-\tau)\ d\tau, \end{aligned}$$

where we have used

$$(A.7) \qquad \partial_t(f(t)\ U(t)) = f'(t)\ U(t) + f(t)\ \delta(t).$$

The prime denotes differentiation with respect to the argument of the function.

STANDARD LAPLACE TRANSFORMS OF SOME BESSEL FUNCTIONS

$f(t)$	$\hat{f}(s)$
$I_0(at);\ t\geq 0$	$\frac{1}{\sqrt{s^2-a^2}}$
$I_0(a\sqrt{t^2+2tb});\ t\geq b\geq 0$	$\frac{1}{\sqrt{s^2-a^2}}\, e^{-b(\sqrt{s^2-a^2}\ -s)}$
$aI_1(at);\ t\geq 0$	$\frac{s}{\sqrt{s^2-a^2}} - 1$
$\frac{ab}{\sqrt{t^2+2tb}}\, I_1(a\sqrt{t^2+2tb});\ t\geq b\geq 0$	$e^{-b(\sqrt{s^2-a^2}\ -s)} - 1$
$\frac{ab}{\sqrt{t^2+2tb}}\, I_1(a\sqrt{t^2+2tb}) + \delta(t);t\geq 0$	$e^{-b(\sqrt{s^2-a^2}\ -s)}$
$\frac{at}{\sqrt{t^2+2tb}}\, I_1(a\sqrt{t^2+2tb});\ t\geq b\geq 0$	$\left(\frac{s}{\sqrt{s^2-a^2}} - 1\right) e^{-b(\sqrt{s^2-a^2}\ -\ s)}$

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A STUDY WAS PERFORMED TO INVESTIGATE THE ELECTROMAGNETIC SCATTERING IN THE TIME DOMAIN BY A PLANE INTERFACE BETWEEN TWO HALF-SPACES. ONE HALF-SPACE IS ASSUMED TO BE VACUUM, WHILE THE OTHER HALF-SPACE IS HOMOGENEOUS AND CONSISTS OF LOSSY MATERIAL. THE INCIDENT FIELD IS ASSUMED TO BE A UNIFORM PLANE WAVE. HENCE, THIS STUDY ADDRESSES THE ONE-DIMENSIONAL SCATTERING PROBLEM.
BOTH HORIZONTAL AND VERTICAL POLARIZATION OF THE INCIDENT FIELD ARE ADDRESSED.
STARTING WITH THE EQUATIONS FOR THE REFLECTED AND TRANSMITTED WAVES IN THE S- OR LAPLACE-DOMAIN, CORRESPONDING TIME-DOMAIN EXPRESSIONS ARE OBTAINED BY APPLYING THE INVERSE LAPLACE TRANSFORM ANALYTICALLY. THESE TIME-DOMAIN RESULTS ARE IN CLOSED FORM, I.E. ARE GIVEN IN TERMS OF ELEMENTARY FUNCTIONS OR INTEGRALS OF ELEMENTARY FUNCTIONS. NUMERICAL RESULTS ARE PRESENTED FOR THE SCATTERING OF A UNIT-STEP AND A NUCLEAR ELECTROMAGNETIC PULSE (NEMP), USING THE DERIVED TIME-DOMAIN EXPRESSIONS. THE NUMERICAL IMPLEMENTATION USES A TIME-MARCHING PROCEDURE.

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